NAVAL POSTGRADUATE SCHOOL
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THESIS

A MINEFIELD RECONNAISSANCE SIMULATION

by

Mauricio Jose Machado Guedes

June 2002

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A MINEFIELD RECONNAISSANCE SIMULATION

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ABSTRACT

The Navy plans to do covert reconnaissance of minefields with a remote underwater vehicle that includes two sensors, one long-range (LR) and one short-range (SR). LR can detect mines, but it cannot distinguish them from harmless mine-like objects. SR can tell the difference, but only by approaching to within short range. A program called MIRES (Minefield Reconnaissance Simulator) is implemented to answer the questions of how the vehicle should perform a search and to estimate the number of mines remaining in the area once the reconnaissance is over. MIRES investigates four modes of search; a planned search with departure to identify an object, a planned search with no departure, and two kinds of random search. It compares these modes of search and identifies the best search mode for a given scenario.

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THESIS DISCLAIMER

The reader is cautioned that the computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any replication of these programs without additional verification is at risk of the user.

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I. INTRODUCTION

A. BACKGROUND

Mines are a menace to naval operations. Conventional methods of minefield reconnaissance operations are expensive and dangerous. In an effort to find a safe and a low-cost solution, the U.S. Navy plans to do covert reconnaissance of minefields with a Long-Term Mine Reconnaissance System (LMRS). The LMRS is a sophisticated autonomous unmanned undersea vehicle (UUV) system that will operate clandestinely from a U.S. Navy nuclear submarine and will be launched and recovered through the submarine's torpedo tubes. The LMRS consists of a self-propelled 21-inch diameter autonomous UUV equipped with mine search and classification sonars for locating objects in a naval operation area of interest. The LMRS provides the early clandestine capability to assess the minefield area and to help commanders decide how to conduct mine countermeasure operations (Ref. [1]).

The UUV has two sensors, one long-range (LR) and one short-range (SR). The LR can detect mines, but it cannot distinguish them from harmless mine-like objects. The SR can tell the difference, but only by approaching to within a short range. The total track length for the UUV is fixed by energy or time constraits, but the question of how the UUV ought to search is still open, as is the question of how to estimate the number of mines remaining in the area once the reconnaissance is over. In particular, the circumstances under which the UUV should depart from a planned track in order to investigate a contact made by the LR is not clear.

B. PROBLEM STATEMENT

This thesis designs and implements a time-step Monte Carlo simulation, called MIRES (Minefield Reconnaissance Simulator), of the reconnaissance of a minefield by the UUV. The specific objectives are

a) To determine the best way for the UUV to search the minefield, and

b) To estimate the number of mines remaining in the minefield after the search.

For item a), the measure of effectiveness (MOE) is the number of mines remaining in the minefield after the search. For the item b), the MOE is the accuracy of the estimate.

Since, in this simulation, mines and mine-like objects are placed at random, multiple runs of the same scenario (i.e., Monte Carlo simulation) are required to generate meaningful performance statistics. Mine-like objects are not mines, but resemble mines closely enough to force the UUV to inspect them, thus reducing its efficiency.

C. TACTICAL QUESTIONS

Washburn (Ref. [2]) considers two basic patterns in searching a minefield. In the first pattern, the UUV follows a planned base track with departures from the base track to identify the object. This is called "planned search". Figure 1 shows an example.

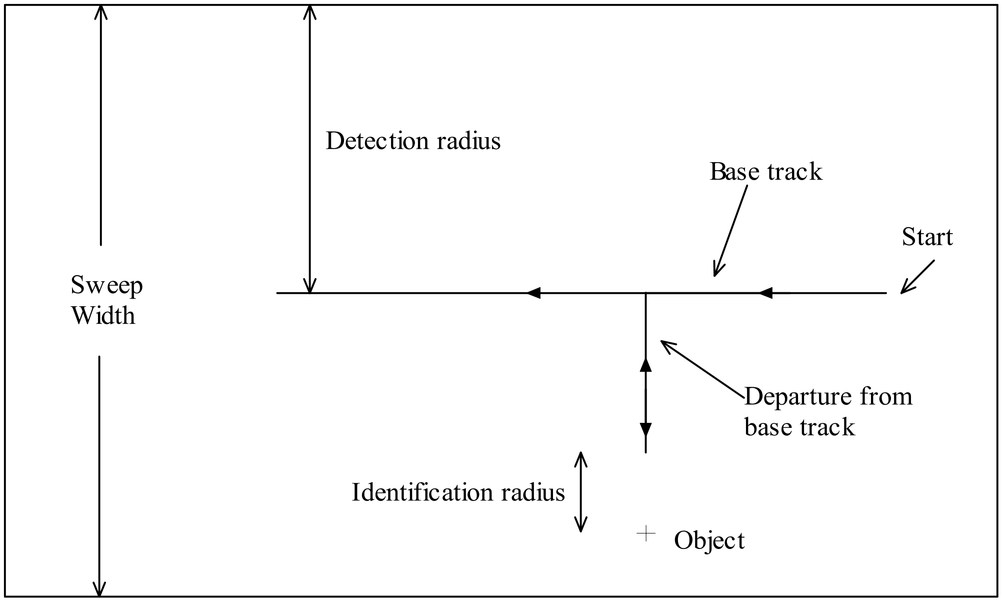

Figure 1: **Planned Search in a Minefield**

In the other pattern, the UUV goes to the nearest detected object until identification is obtained. After identification, the UUV heads toward the next nearest object, or if there are no objects detected, it maintains the last direction until a detection is obtained. If the UUV reaches the edges of the minefield, it bounces back randomly with an angle ϕ to the minefield. This is called “random search”. Figure 2 shows an example.

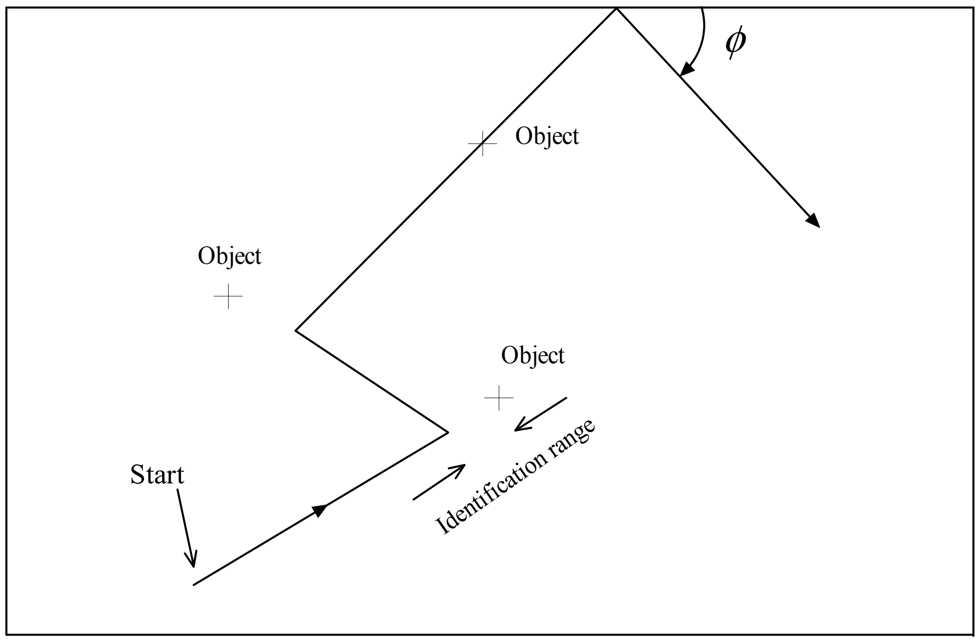

Figure 2: Random Search

The LR and the SR are considered cookie-cutter sensors, that is, sensors that have detection probability of "one" when the target is inside the sensor 's detection range and "zero" otherwise.

II. METHODOLOGY

A. SYSTEM DESIGN

MIRES is a time-step simulation. The idea is to divide the simulated time into intervals of the same length and to recalculate all model variables at the end of each of these intervals

MIRES is designed to allow the user to define tactical parameters and to simulate the search path in a rectangular two-dimensional minefield. All parameters are written in an input file. The design of the system is presented schematically in Figure 3.

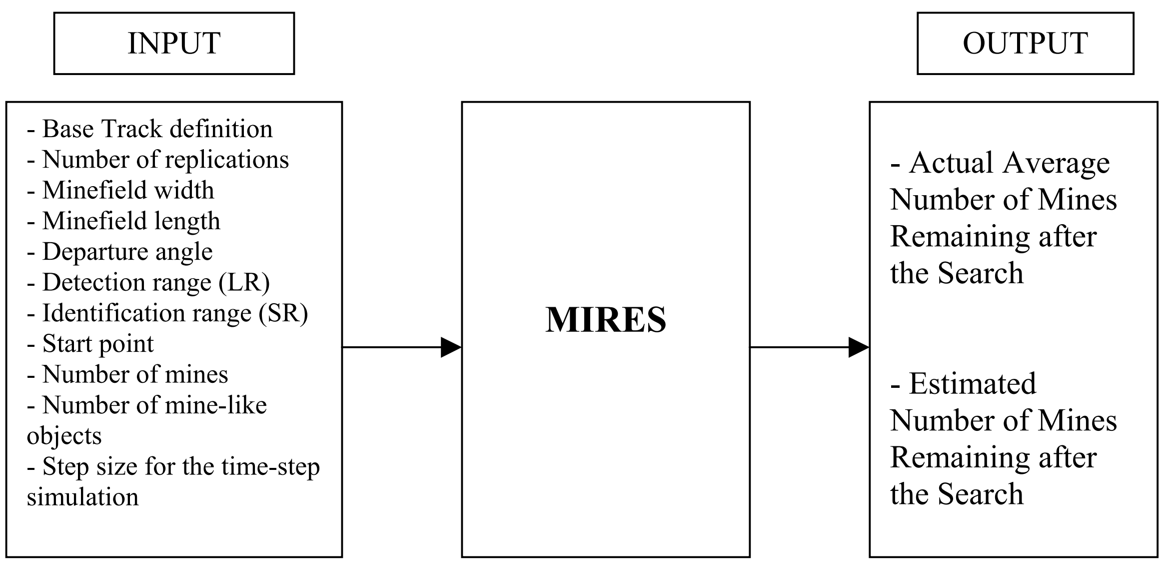

Figure 3: System Design

MIRES sets the number of mines and mine-like objects and then disposes them in the minefield. From the starting point, the program begins the search according to the chosen mode, keeping track of the track length and the number of mines detected and identified. The model considers the area covered by the UUV (the minefield area searched by the UUV LR sonar), the distance traveled by the UUV, the number of mines

and the number of mine-like objects identified, and the number of objects detected but not identified. The number of mines remaining in the minefield is estimated by using the maximum likelihood estimator and compared with the actual number of mines remaining. At the end of the simulation, the program calculates the output statistics (average and 95% confidence interval) of the above quantities and writes them in a file.

B. MINEFIELD DEFINITION

The minefield is defined as a X-Y coordinate system with the X coordinates representing the horizontal direction and the Y coordinates representing the vertical direction as shown in Figure 4.

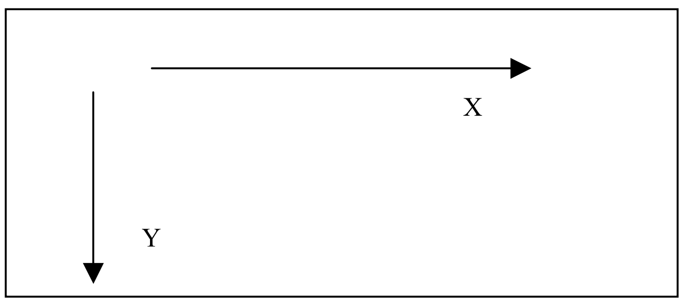

Figure 4: X-Y Coordination System

The minefield for the simulation is a two-dimensional field sized 635 units by 960 units. One pixel is one square unit. The objects are located at random, subject only to the constraint that the average number of mines should be *N*, and the average number of mine-like objects should be $\overline{N}$, both defined by the user. This characterizes a Poisson field. So, the average total number of objects in the minefield is $N_{tot} = N + \overline{N}$. An object occupies one pixel. So, the minefield has 635 x 960 = 609,600 possible spots for the

objects. The objects are represented in the coordinate system as shown in Figure 5. In this example, object 1 has coordinates (2,4), and object 2 has coordinates (6,5).

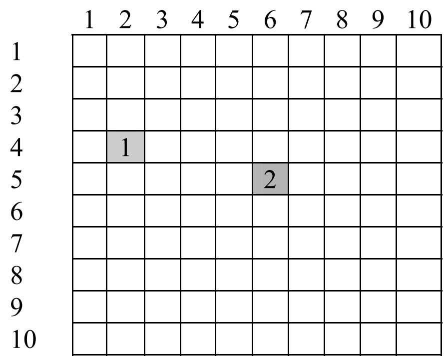

Figure 5: MIRES Schematic Minefield Representation

In the simulation, a Poisson distribution determines the number of mines and the number of mine-like objects randomly, with a mean established by the user in each case. The position of each object in the minefield is determined randomly using a bi-variate Uniform distribution. An example of a minefield with 200 objects is shown in Figure 6.

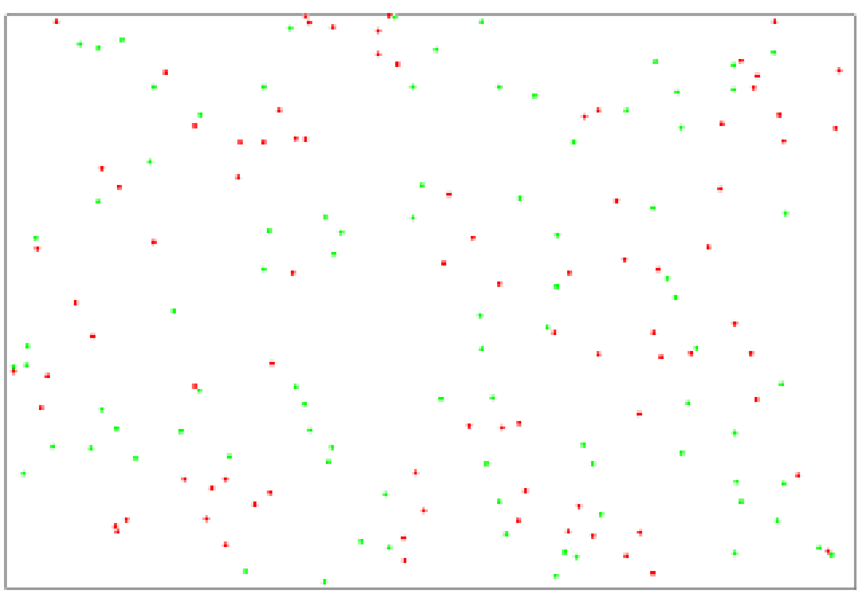

Figure 6: An Example of Objects Randomly Distributed in a Minefield

C. SEARCH PATTERN

Two distinct search patterns are modeled. The first is a planned search that uses a systematic search in strips, with equal and predetermined spaces between paths; the second is a random search that searches at random, starting from an initial position following an initial direction. Furthermore, two kinds of random search are investigated, with the UUV having memory in one and no memory in the other.

1. Planned Search

From an initial point in the minefield, the search is done in a straight line until the UUV detects an object. When the detection happens, the UUV departs from the base track to investigate the contact. The UUV can depart from the base track in a preset angle θ chosen by the user, where 0° ≤ θ ≤ 90°. Once the object is identified, the UUV returns to the base track and resumes the search. Figure 7 shows an example of a base track. M is the distance from the edges of the minefield and S is the track spacing.

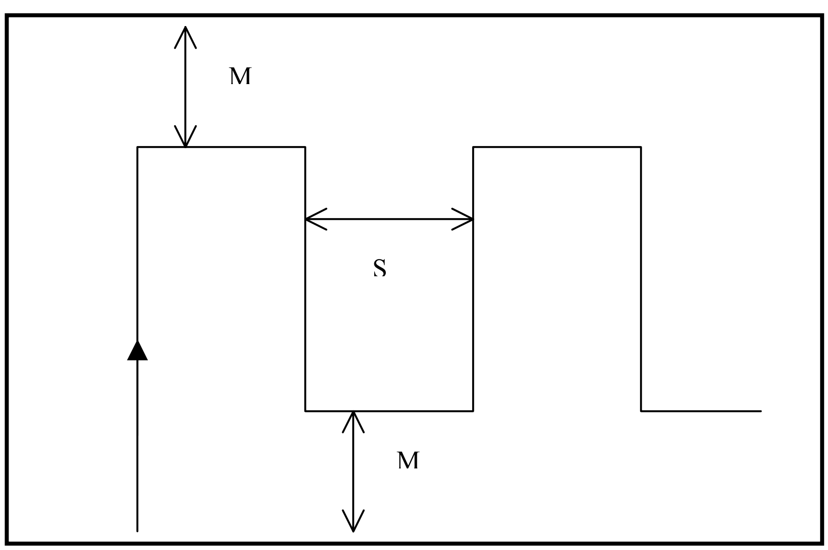

Figure 7: Base Track Followed by the UUV

An example of a planned search in the minefield is shown in Figure 8, part of the graphical output from MIRES, shown here in black and white. In this case, the departure angle and the identification range are set equal to zero. An exception occurs in corners. From corners, the UUV doesn't take into account the departure angle.

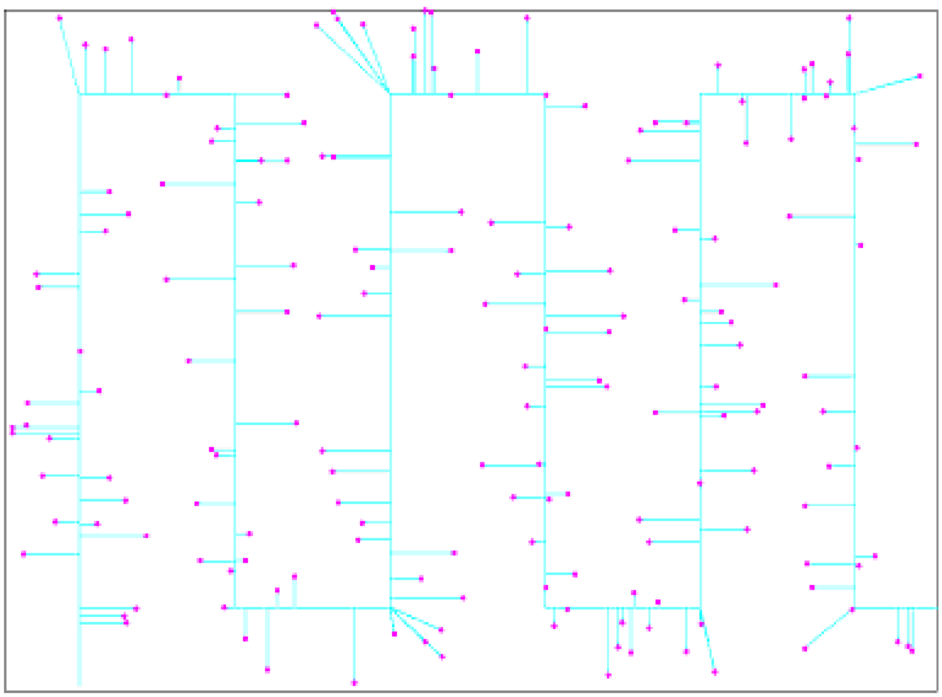

Figure 8: Example of Planned Search

2. Random Search with Memory

From an initial position and assuming an initial direction, the UUV moves in a straight line until it detects an object. After detection the UUV turns to point toward the object and approaches until the object is identified. After that, the UUV heads to the nearest object detected but not identified, or, if there are no objects detected, the UUV continues in a straight line until an object is detected. When it reaches an edge, the UUV bounces back with an angle ϕ as shown in Figure 9. In all cases ϕ is taken to be uniform in the interval from 0° to 90°, the angle from the local boundary to the reflected path.

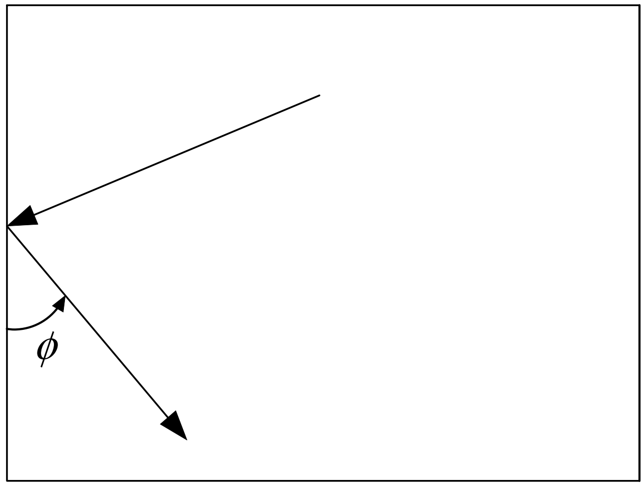

Figure 9: Example of Bounce

The UUV keeps all contacts in memory, even those contacts beyond the present UUV detection range. Figure 10 shows an example of a random search with memory, on which all objects were identified.

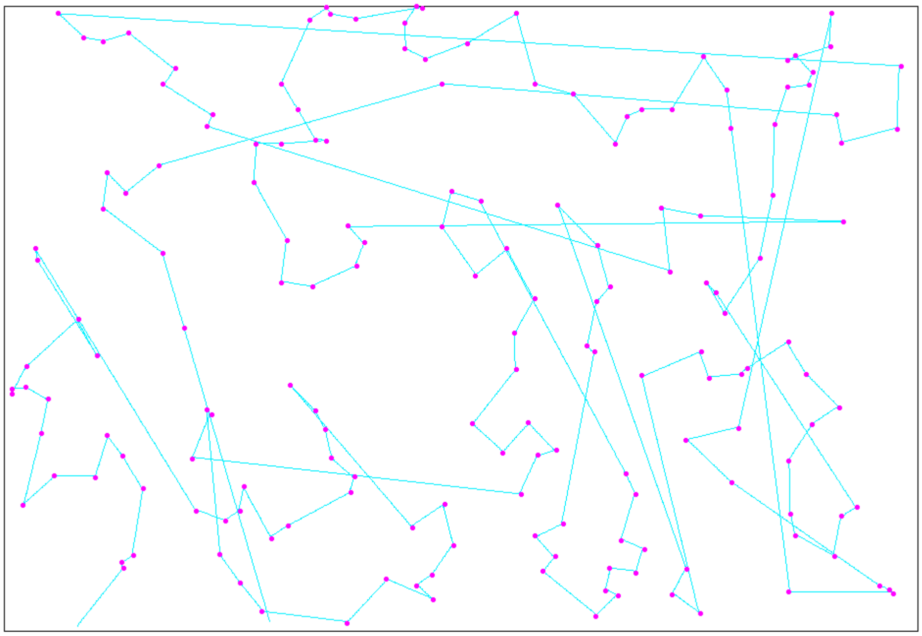

Figure 10: An Example of a Random Search

3. Random Search with No Memory

This is essentially the same as the prior case, but now the UUV considers only the objects inside its current detection range in determining its next maneuver.

D. TRACK LENGTH CALCULATION

The position of the UUV is calculated as follows:

$$x_{new} = x_{old} * \cos(\alpha) * inc$$

$$y_{new} = y_{old} * \sin(\alpha) * inc,$$

where α is the UUV direction, and *inc* is the length of the time-step advance.

The *i*th time the UUV advances in its path, the distance from the old position to the new position is calculated using the formula $D_i = \sqrt{(x_{new} - x_{old})^2 + (y_{new} - y_{old})^2}$, where (x_{new}, y_{new}) are the UUV coordinates for the new position, after the advance, and (x_{old}, y_{old}) are the UUV coordinates before the advance. The total distance traveled is calculated by $D_{total} = \sum_{i=1}^{n} D_i$, where n is the total number of time-steps that the UUV has advanced.

E. AREA COVERED CALCULATION

The smallest portion of the minefield considered in calculating the area covered is a segment formed by nine pixels. Despite the loss of some precision, avoiding the consideration of every pixel allows the simulation to run faster. MIRES keeps a matrix, initially clear, that represents all segments in the minefield. These segments are used to represent the area covered by the UUV as it advances. In each advance, all segments inside or partially inside the present UUV LR sonar detection range are set to 1. Once set,

the position is not cleared until the end of the run. After the end of the run, the matrix will have squares covered by the UUV LR sonar set to 1. Figure 11 shows an example of how the matrix is set. It shows 9 squares formed by 81 pixels. Each square formed by nine pixels is a position in the matrix of area covered. The area shaded indicates that that segment was covered by the LR sonar, and the curve indicates the LR sonar range.

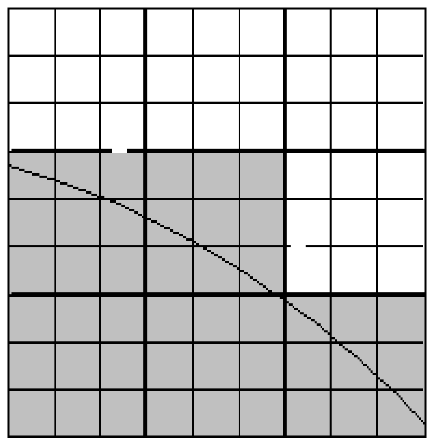

Figure 11: Example of Setting the Matrix of Area Covered

F. ESTIMATION OF THE NUMBER OF MINES REMAINING

After finishing the search, the values for the number of mines, mine-like objects and contacts detected but not identified will be known. Based on the number of objects detected and on the fraction of mines identified, estimating the number of mines remaining in the minefield after finishing the search is possible.

Let:

N_{found} = Total number of objects found (detected + identified)

N_{ident} = Number of objects identified (mines + mine-like objects)

N_{mi} = Number of mines identified

$\overline{N}_{mi}$ = Number of mines remaining

A_{co} = Area covered by the UUV

A_{unco} = Area remaining not covered by the UUV

β = Fraction of objects that are mines

The estimator of fraction of objects that are mines is

$$\hat{\beta} = \frac{N_{mi}}{N_{ident}}$$

And the estimator for the mines remaining in the minefield after the reconnaissance is

$$\hat{\overline{N}}_{mi} = \left(N_{found}\right)\left(\frac{A_{unco}}{A_{co}}\right)\left(\hat{\beta}\right) + \left(N_{found} - N_{ident}\right)\hat{\beta}$$

The first term estimates mines in the uncovered area, and the second term estimates mines in the covered area.

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III. SOFTWARE

A. STRUCTURE

MIRES is written in FORTRAN 95, using structured programming. A graphic visualization of the simulation was developed (see Figure 8) to keep verify the code. The validation of this simulation is beyond of the scope of this work.

The MIRES source code can be obtained by sending an e-mail to the author at mauricio532@hotmail.com

The program has the same general structure for the planned and random search modes. The following flow chart (Figure 12) shows the general structure. The "Conduct search" block depends on the type of search being conducted.

B. LOGIC OF THE SEARCH USING THE RANDOM PATTERN

The UUV goes to the nearest object detected. After identification, it goes to another nearest object detected, and so on. If there are no objects detected, the UUV keeps the last direction until an object is detected. The flow chart in Figure 13 describes the logic of the random search with memory, and the flow chart Figure 14 describes the logic of the random search with no memory.

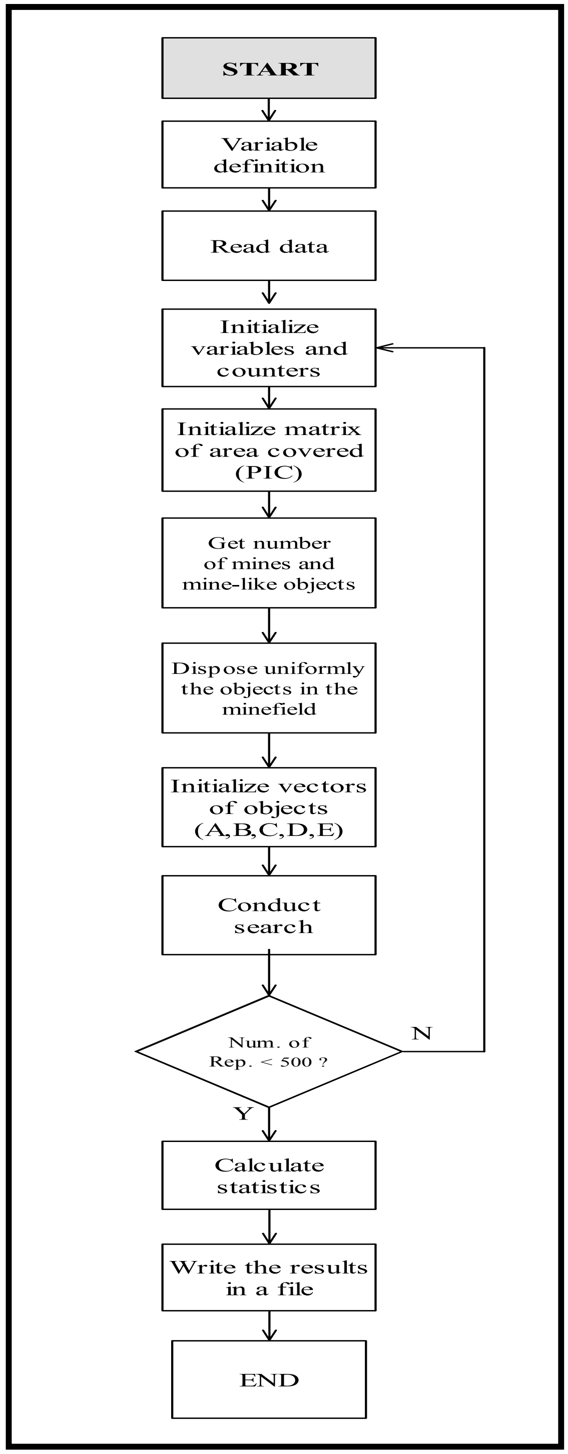

Figure 12: Flow Chart of the General Structure of MIRES

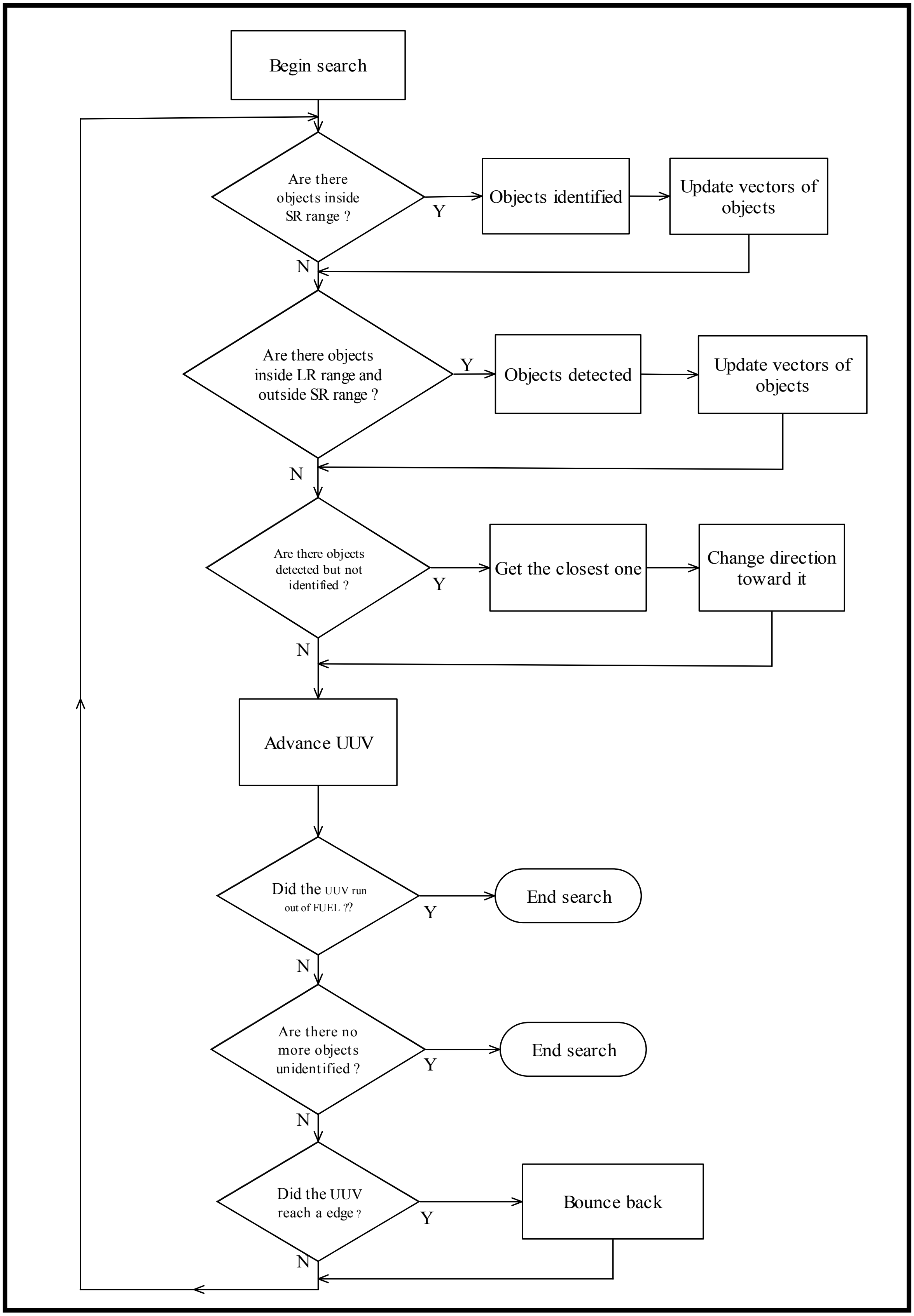

Figure 13: Logic of the Random Search with Memory

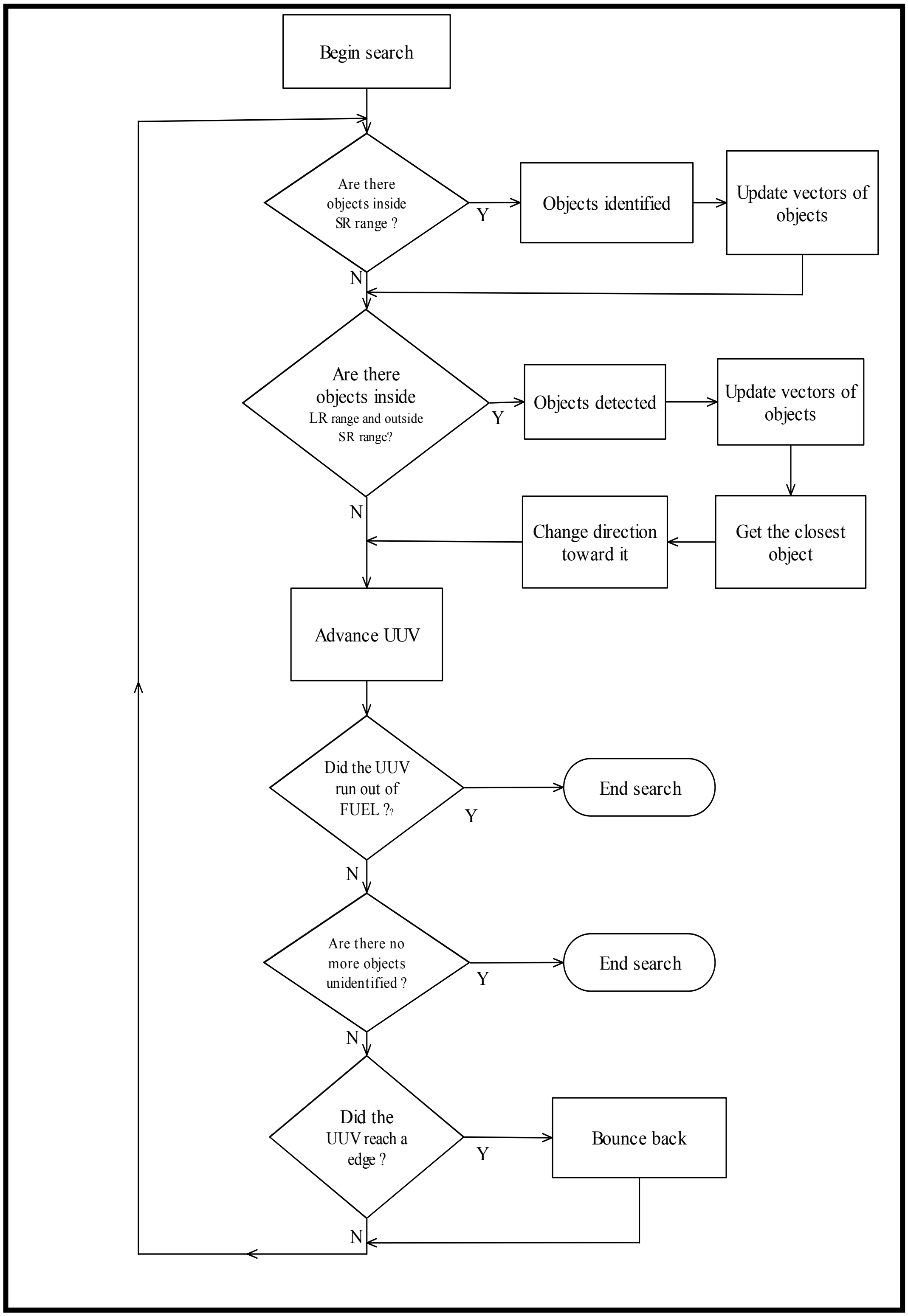

Figure 14: Logic of the Random Search with No Memory

C. LOGIC OF THE SEARCH USING THE PLANNED PATTERN

Vertical and horizontal straight lines, called tracks, form the planned search. The UUV searches from left to right, from bottom to top. The user defines the track by defining the track spacing S and the distance M from the edges at which the UUV turns. See Figure 7.

Vertical and horizontal segments, as many as necessary to cover the minefield (limited by the distance available to search), form the base track.

When the UUV is near of the end of a segment, it defers identification of an object that is nearer to the next segment. Figure 15 shows an example. The object is first detected in the vertical segment, but the object is more easily reached from the next (horizontal) segment. So, the UUV departs to investigate the object from the horizontal segment. The objects located on the right of the dotted line are closer to the horizontal segment.

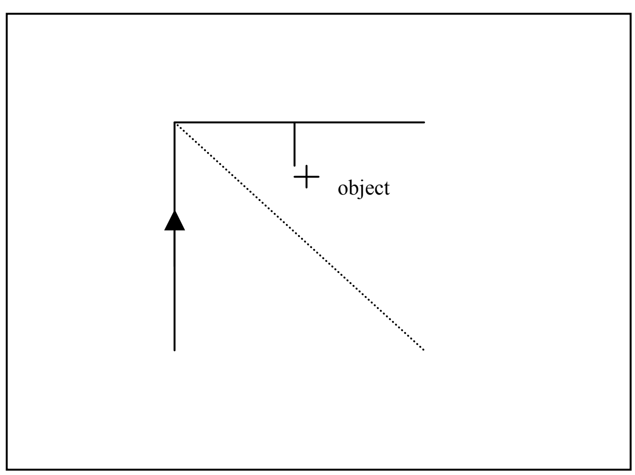

Figure 15: Behavior Close to a Corner

The UUV departs to investigate a contact if the object is inside of the area reachable by the UUV. Figure 16 shows an example of area reachable from a vertical segment, defined by ABCDEF. The angle θ is the departure angle.

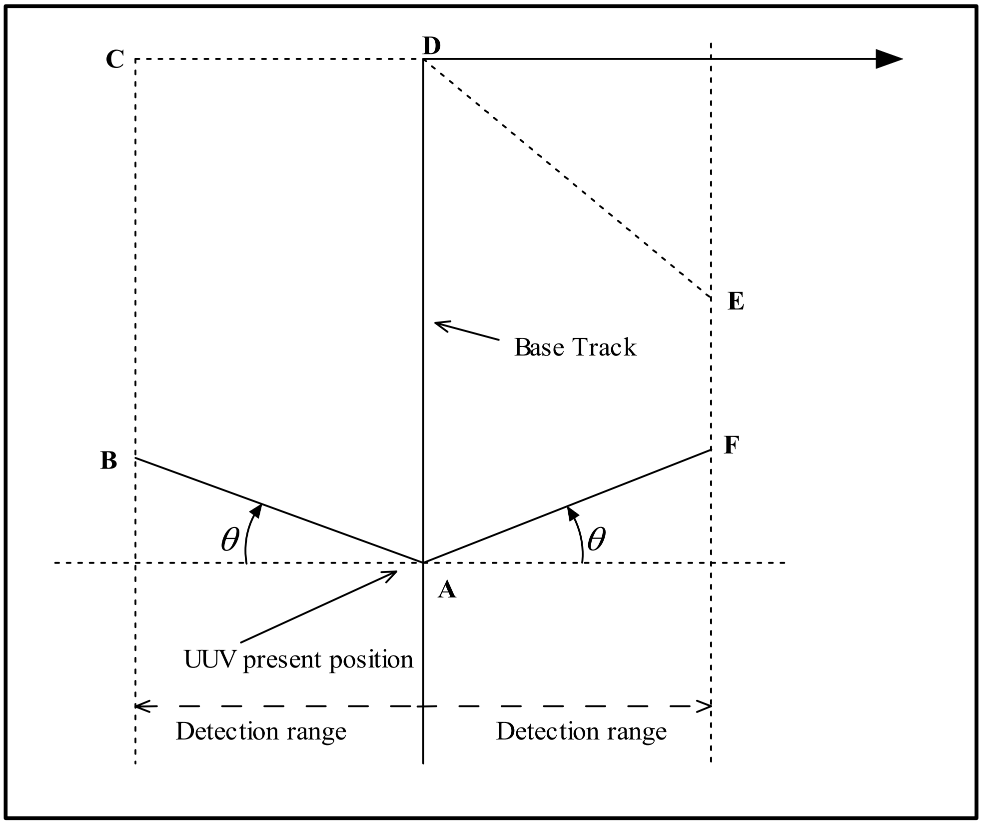

Figure 16: Definition of Area Reachable

From each corner (change of direction), the UUV goes to each object located in the square defined M × S/2, as shown in Figure 17 for some corners. The UUV goes to the object, identifies it and comes back following the same trajectory, repeating this procedure for every detected object in the square. From a corner, the departure angle θ is not taken in account.

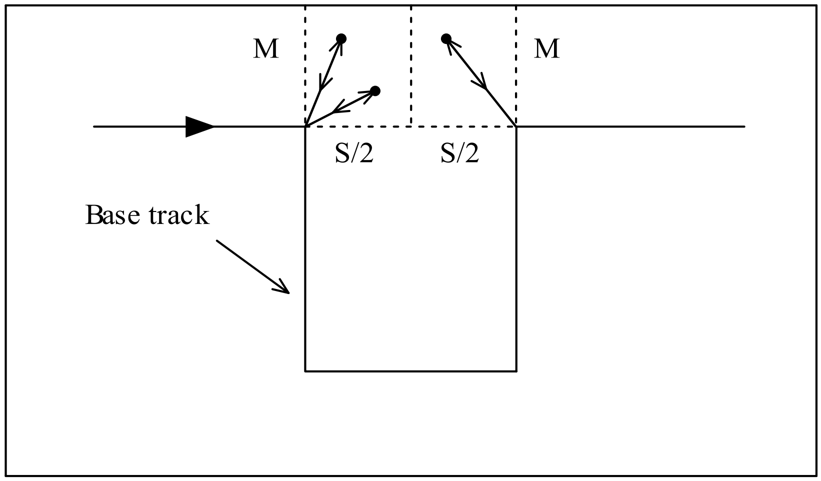

Figure 17: Going to the Objects from the Corners

The following flow chart (Figure 18) decribes the logic of the search using the planned pattern.

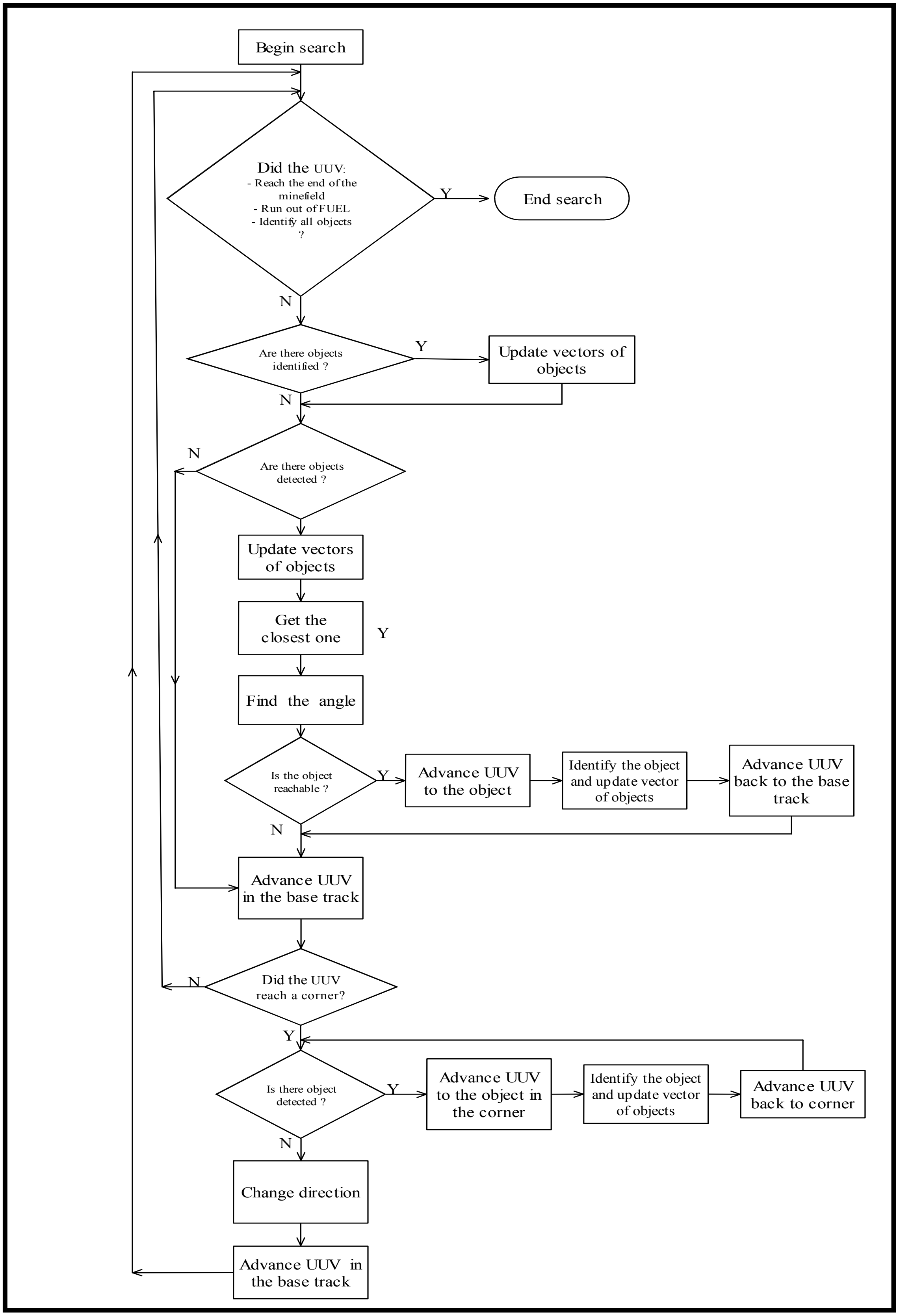

Figure 18: Logic of the Planned Search

IV. SELECTION OF THE BEST MODE TO SEARCH

A. EXHAUSTIVE SEARCH

Exhaustive search will be used to find an upper bound for the time to search the minefield. Two cases are considered; a low-density minefield and a high-density minefield. The optimum track space used is given by the formula, from Reference [2]:

$$S = \sqrt{2A/N} \qquad (1)$$

where S is the track space, A is the area of the minefield, and N is the average number of objects. For this simulation, using formula (1), the optimum track space for a minefield with a low density of mines is 156 units, and for a minefield with a high density of mines the optimum track space is 78 units. With this track spacing, the average track length needed to exhaustively search the region by visiting each object is, in theory, DIST=$\sqrt{2AN}$ (Ref. [2]). For the high-density minefield, this distance is $\sqrt{2(960)\times(635)\times(200)} = 15,615$, or 7,808 for the low-density minefield.

B. TIME CONSTRAINED SEARCH

When the time available for the search permits only a distance less than DIST, the search is called time-constrained, and exhaustive search may not be the best way to search. So, it is necessary to compare other search modes to find the best search mode, taking in account the minefield density, detection range and the time available to search.

C. PARAMETER SETTING

Table 1 lists the simulation parameters for the cases studied in the sequel. The identification range is assumed to be 20% of the detection range.

Table 1: Simulation Parameters

Parameter	Setting
Number of mine-like objects	Low: 25
	High: 100
Number of mines	Low: 25
	High: 100
Minefield width	960 units
Minefield height	635 units
Detection range	Small: 20 units
	Medium: 40 units
	Large: 80 units
Identification range	Small: 4 units
	Medium: 8 units
	Large: 16 units
Track space (S) (planned mode)	Small: 40 units
	Medium: 80 units
	Large: 160 units
Initial position (x,y)	Small: (20,635)
	Medium: (40,635)
	Large: (80,635)
Time-step advance	3 units
Departure angle θ (planned mode)	0° or 20°
Vertical distance M from edges (planned mode)	S/2
Number of iterations	500
Distance available to search	80% of DIST 60% of DIST 40% of DIST

D. TEST CONDUCTED

There are four search modes. They are:

1 - Planned search with 0 degree as departure angle

2 - Planned search with 20 degrees as departure angle

3 - Random search with memory.

4 - Random search with no memory.

These search modes are tested in different settings of detection range and identification range, distance available to search and minefield density. Table 2 shows the different search settings.

Table 2: Search Settings

Detection range	Small = 20 units Medium = 40 units Large = 80 units
Distance available	80% of DIST 60% of DIST 40% of DIST
Minefield density	Low = 50 objects High = 200 objects

To find the best way of searching, an experiment is performed using all combinations of the minefield and UUV settings, a total of 18 different scenarios. For each search mode, each of these scenarios is replicated 500 times. This number gives a good simulation time response and a reasonable confidence interval.

E. RESULTS

First MIRES is used to calculate the DIST required to conduct an exhaustive search in a low-density minefield and in a high-density minefield. Running 500 replications using mode 1 search with SR range set to zero does it. Table 3 presents the average DIST for a nearly exhaustive search.

Table 3: DIST for Nearly Exhaustive Search

	DIST	
Mode	Low density	High density
1	**6535**	**13978**

Why nearly? Due to its circular detection area, the UUV doesn't cover small regions in the corners and between the vertical segments close to the edge, as can be seen in Figure 19. The uncovered regions are shaded.

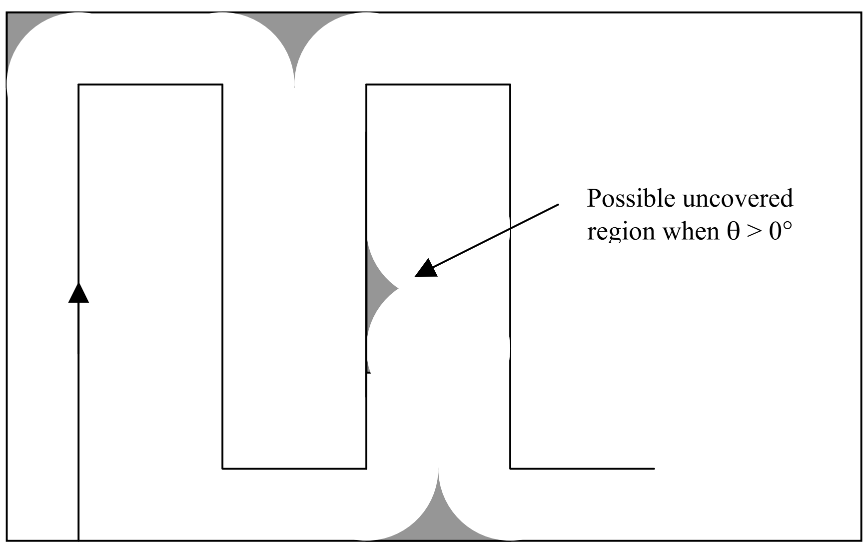

Figure 19: Area not Covered by the UUV

Any non zero departure angle also may result in small, uncovered regions, as illustrated.

Because of these small areas not covered by the UUV and some cleverness built into MIRES (like the one described in Figure 15), the distances found for exhaustive search are lower than the theoretical results.

1. Results When Distance Available to Search is 80 % of DIST

Table 4 shows the actual average number of mines remaining in the minefield after the search (avg) and its standard deviation (sd).

As the maximum distance available to search is 80 % of the average distance required to complete the track (DIST), in some runs the UUV doesn't complete the track.

When a small LR detection range is used in a low density minefield, the mode used is quite insignificant. As the detection range increases, the differences become more evident. For a high-density minefield, the poor performance of mode 1 in the medium and large detection ranges compared with mode 2 is because mode 1 doesn't complete the base track in most of runs.

In a high-density minefield, modes 3 and 4 become better than modes 1 and 2 as the LR detection range gets larger. Actually, it is expected, from the results, that when the detection range gets larger than half of the optimum track space for a specific minefield, modes 3 and 4 will perform better than modes 1 and 2.

Table 4: Actual Average Number of Mines Remaining for 80 % of DIST

	Actual Average Number of Mines Remaining											
	Low Density						High Density					
	Small Detec. Range		Medium Detec. Range		Large Detec. Range		Small Detec. Range		Medium Detec. Range		Large Detec. Range	
Mode	avg	sd	Avg	sd	avg	sd	avg	sd	avg	sd	avg	sd
1	**17.2**	4.1	**10.8**	3.3	**4.5**	2.4	**42.1**	6.3	**17.1**	5.2	**15.7**	6.5
2	**17.2**	4.1	**10.4**	3.2	**2.7**	1.9	**40.0**	6.2	**9.0**	4.3	**7.1**	5.2
3	**17.4**	4.1	**12.6**	3.5	**6.6**	3.2	**49.0**	7.7	**25.6**	8.0	**2.5**	3.3
4	**17.4**	4.1	**12.7**	3.7	**6.3**	3.0	**48.7**	7.2	**25.7**	7.3	**4.9**	5.0

Table 5 shows the MIRES estimates of mines remaining after the end of the search (avg) and the 95% confidence interval widths (w).

Table 5: Estimated Number of Mines Remaining for 80 % of DIST

	Estimated Number of Mines Remaining											
	Low Density						High Density					
Mode	Small Detec. Range		Medium Detec. Range		Large Detec. Range		Small Detec. Range		Medium Detec. Range		Large Detec. Range	
	Avg	w	Avg	W	avg	w	avg	w	Avg	w	avg	w
1	**17.4**	1.12	**11.3**	0.59	**4.8**	0.42	**42.0**	1.10	**17.4**	0.83	**15.9**	1.06
2	**17.2**	1.10	**10.6**	0.53	**3.0**	0.31	**40.0**	1.02	**9.3**	0.67	**7.7**	0.88
3	**17.4**	1.12	**12.9**	0.67	**6.6**	0.44	**49.1**	1.34	**26.0**	1.23	**2.7**	0.47
4	**17.4**	1.12	**12.9**	0.67	**6.5**	0.42	**48.7**	1.29	**25.6**	1.15	**5.2**	0.80

The estimation bias *b* is the difference between the expected values of the estimates and the actual number of mines remaining, E(Y) - E(X) = *b*, where

X: actual number of remaining mines random variable.

Y: estimated number of mines remaining random variable.

The estimator for the bias *b* is

$$\hat{b} = \frac{\sum_{i=1}^{500}\left(Y_i - X_i\right)}{500}$$

Table 6 shows the bias of the estimated values.

Table 6: Bias of MIRES Estimation for 80 % of DIST

	Bias					
	Low Density			High Density		
Mode	Small Detec. Range	Medium Detec. Range	Large Detec. Range	Small Detec. Range	Medium Detec. Range	Large Detec. Range
1	0.2	0.5	0.3	−0.1	0.3	0.2
2	0	0.2	0.3	0	0.3	0.6
3	0	0.3	0	0.1	0.4	0.2
4	0	0.2	0.2	0	−0.1	0.3

The average bias is 0.30. For the null hypothesis H_0: the true bias across all 24 scenarios is zero, the null hypothesis cannot be rejected at the 0.01 level. This figure is based entirely on the data in Table 6.

2. Results When Distance Available to Search is 60 % of DIST

Table 7 shows the actual average number of mines remaining in the minefield after the search (avg) and its standard deviation (sd).

Now the difference noticed between modes 1 and 2 in a high density minefield when LR detection range is medium and large is not so evident, because now both modes could not finish the base track.

Again modes 3 and 4 performed better when the LR detection range is large in a high-density minefield.

Table 7: Actual Average Number of Mines Remaining for 60 % of DIST

	Actual Average Number of Mines Remaining											
	Low Density						High Density					
	Small Detec. Range		Medium Detec. Range		Large Detec. Range		Small Detec. Range		Medium Detec. Range		Large Detec. Range	
Mode	avg	sd	avg	sd	avg	sd	avg	sd	avg	sd	avg	sd
1	**19.1**	4.3	**14.4**	3.8	**9.3**	3.2	**56.5**	7.4	**37.3**	6.6	**34.8**	7.4
2	**19.1**	4.3	**14.0**	3.6	**7.9**	2.9	**54.8**	7.2	**31.2**	6.0	**26.9**	7.4
3	**19.3**	4.5	**15.2**	3.8	**9.6**	3.7	**58.8**	7.9	**37.8**	9.3	**6.5**	5.2
4	**19.4**	4.5	**15.0**	3.9	**9.3**	3.8	**58.7**	8.3	**37.0**	8.8	**12.2**	8.3

Table 8 shows the MIRES estimate of mines remaining after the end of the search (avg) and its 95% confidence interval width (w).

Table 8: Estimated Number of Mines Remaining for 60 % of DIST

	Estimated Number of Mines Remaining											
	Low Density						High Density					
Mode	Small Detec. Range		Medium Detec. Range		Large Detec. Range		Small Detec. Range		Medium Detec. Range		Large Detec. Range	
	avg	w	avg	w	avg	w	Avg	w	avg	w	avg	w
1	**19.2**	1.50	**14.7**	0.84	**9.7**	0.61	**56.0**	1.61	**37.6**	1.20	**35.0**	1.35
2	**19.2**	1.48	**14.2**	0.80	**8.3**	0.52	**54.6**	1.54	**31.4**	1.03	**27.3**	1.30
3	**19.3**	1.49	**15.0**	0.88	**9.5**	0.58	**58.5**	1.69	**37.4**	1.39	**6.7**	0.80
4	**19.3**	1.49	**14.9**	0.86	**9.4**	0.55	**58.1**	1.63	**37.2**	1.39	**12.4**	1.39

Table 9 shows the bias of the estimated values.

Table 9: Bias of MIRES Estimation for 60 % of DIST

	Bias					
	Low Density			High Density		
Mode	Small Detec. Range	Medium Detec. Range	Large Detec. Range	Small Detec. Range	Medium Detec. Range	Large Detec. Range
1	–0.1	–0.3	0.4	0.5	–0.3	0.2
2	–0.1	–0.2	–0.4	0.2	–0.2	–0.4
3	0	0.2	0.1	0.3	0.4	–0.2
4	0.1	0.1	–0.1	0.6	–0.2	–0.2

The average bias is 0.02. For the null hypothesis H_0, the null hypothesis cannot be rejected at the 0.01 level. This figure is based entirely on the data in Table 9.

3. Results when distance available to search is 40 % of DIST

Table 10 shows the actual average number of mines remaining in the minefield after the search (avg) and its standard deviation (sd).

Here, mode 4 is better when the LR detection range is medium in a high density minefield. The others results follow the same pattern of the two prior scenarios.

Table 10: Actual Average Number of Mines Remaining for 40 % of DIST

	Actual Average Number of Mines Remaining											
	Low Density						High Density					
	Small Detec. Range		Medium Detec. Range		Large Detec. Range		Small Detec. Range		Medium Detec. Range		Large Detec. Range	
Mode	avg	sd	Avg	sd	avg	sd	avg	sd	avg	sd	avg	sd
1	**21.0**	4.6	**17.8**	4.3	**14.2**	4.0	**70.5**	8.3	**57.5**	7.7	**54.6**	8.0
2	**21.0**	4.6	**17.5**	4.1	**13.2**	3.8	**69.5**	8.2	**53.4**	7.3	**49.2**	7.9
3	**21.0**	4.6	**18.7**	4.4	**14.7**	4.0	**70.1**	8.2	**53.4**	8.9	**24.7**	7.7
4	**21.0**	4.6	**18.7**	4.4	**14.2**	4.0	**69.4**	8.9	**52.2**	8.9	**26.6**	9.5

Table 11 shows the MIRES estimate of mines remaining after the end of the search (avg) and its 95% confidence interval width (w).

Table 11: Estimated Number of Mines Remaining for 40 % of DIST

	Estimated Number of Mines Remaining											
	Low Density						High Density					
	Small Detec. Range		Medium Detec. Range		Large Detec. Range		Small Detec. Range		Medium Detec. Range		Large Detec. Range	
Mode	avg	w	avg	w	avg	w	avg	w	avg	w	avg	w
1	**21.2**	2.01	**18.3**	1.31	**14.7**	0.92	**70.4**	2.39	**57.8**	1.81	**54.6**	1.90
2	**21.1**	1.98	**17.8**	1.24	**13.7**	0.84	**69.3**	2.29	**53.6**	1.61	**49.5**	1.78
3	**21.2**	2.01	**18.9**	1.33	**14.8**	0.90	**70.5**	2.36	**54.1**	1.81	**24.8**	1.16
4	**21.2**	2.01	**18.6**	1.34	**14.5**	0.87	**71.0**	2.28	**52.3**	1.71	**27.4**	1.62

Table 12 shows the bias of the estimated values.

Table 12: Bias of MIRES Estimation for 40 % of DIST

	Bias					
	Low Density			High Density		
Mode	Small Detec. Range	Medium Detec. Range	Large Detec. Range	Small Detec. Range	Medium Detec. Range	Large Detec. Range
1	0.2	0.5	0.5	–0.1	0.3	0
2	0.1	0.3	0.5	–0.2	0.2	0.3
3	0.2	0.2	0.1	0.4	0.7	0.1
4	0.2	–0.1	0.3	0.6	0.1	0.8

The average bias is 0.26. For the null hypothesis H_0, the null hypothesis is rejected at the 0.01 level. This figure is based entirely on the data in Table 12.

V. CONCLUSIONS

A. ESTIMATE OF MINES REMAINING IN THE MINEFIELD

Despite some bias at 40% of DIST, MIRES was able to make good predictions of the number of mines remaining in the minefield.

B. BEST MODE TO SEARCH

The best mode to search in each scenario is given in the Table 13.

Table 13: Best Mode to Use to Search the Minefield

	Low Density			High Density		
DIST	Small Det. Range	Medium Det. Range	Large Det.Range	Small Det. Range	Medium Det. Range	Large Det. Range
80 %	**1 or 2**	**2**	**2**	**2**	**2**	**3**
60 %	**1 or 2**	**2**	**2**	**2**	**2**	**3**
40 %	**1, 2, 3 or 4**	**2**	**2**	**2**	**4**	**3**

The following conclusions were obtained from the Table 13:

a) As the detection range gets larger, mode 3 (random search with memory) becomes the best mode with which to search. This is reasonable, because when the detection range becomes larger, the UUV is able to cover big areas of the minefield, detecting more mines and because of this wasting less time traveling without a contact. It can be seen comparing Figure 20 and Figure 21. These figures were obtained using a high density minefield, large detection range (80 units) and 80 % of DIST.

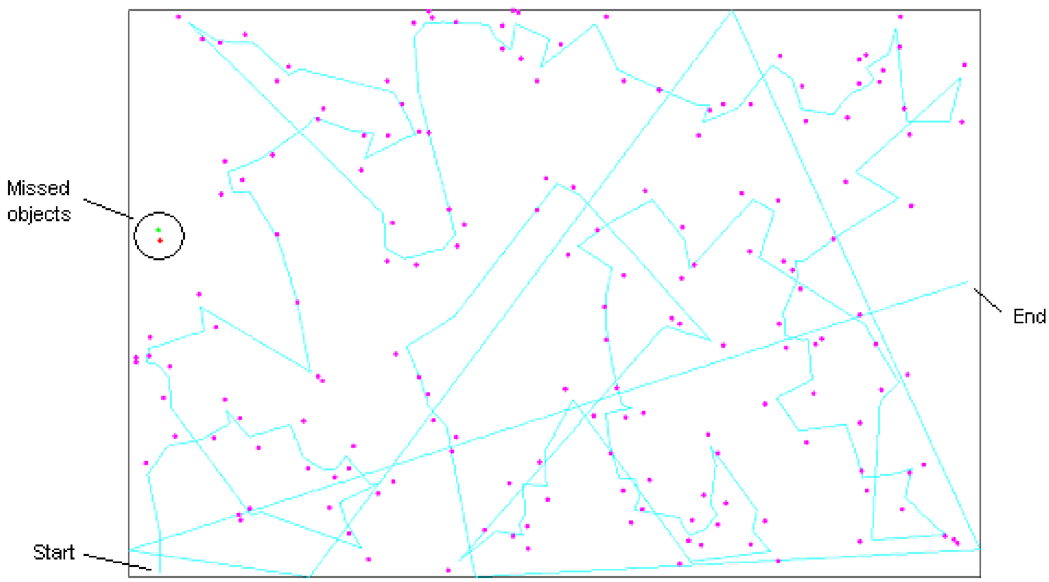

Figure 20: Random Search with Memory

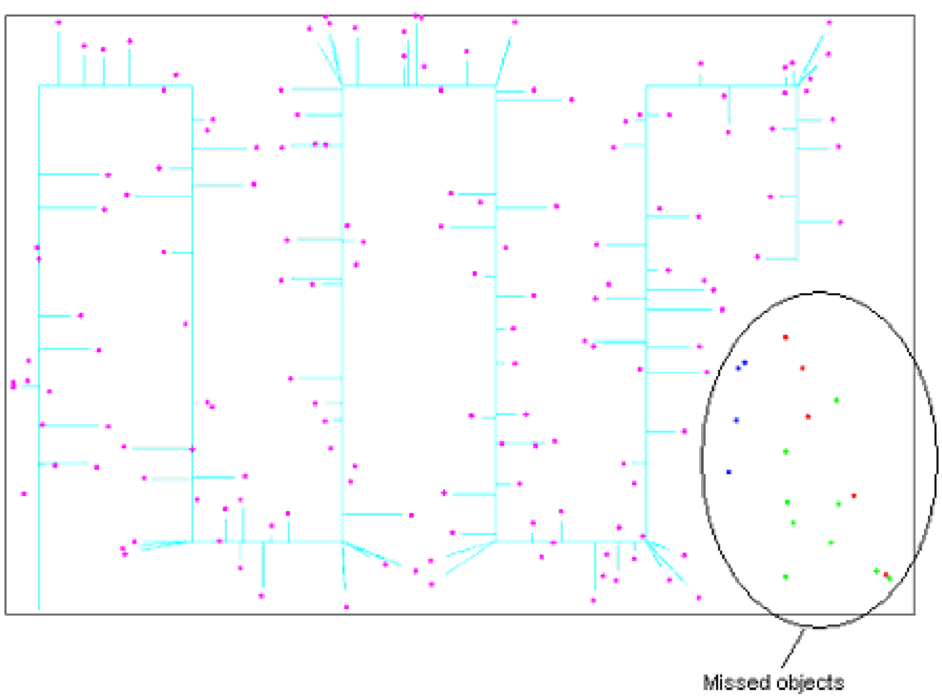

Figure 21: Planned Search with 0° as Departure Angle

In Figure 20, it can be seen that only two objects were not detected, while in Figure 21, several objects were not detected because the base track was not completed.

b) Mode 2 (planned search with 20° as departure angle) is best in the majority of the scenarios. Mode 2 is never worse than mode 1. The departure angle of 20° allows the UUV to go farther, and in some replications, the UUV managed to reach the end of the search, as is shown in the Figure 22.

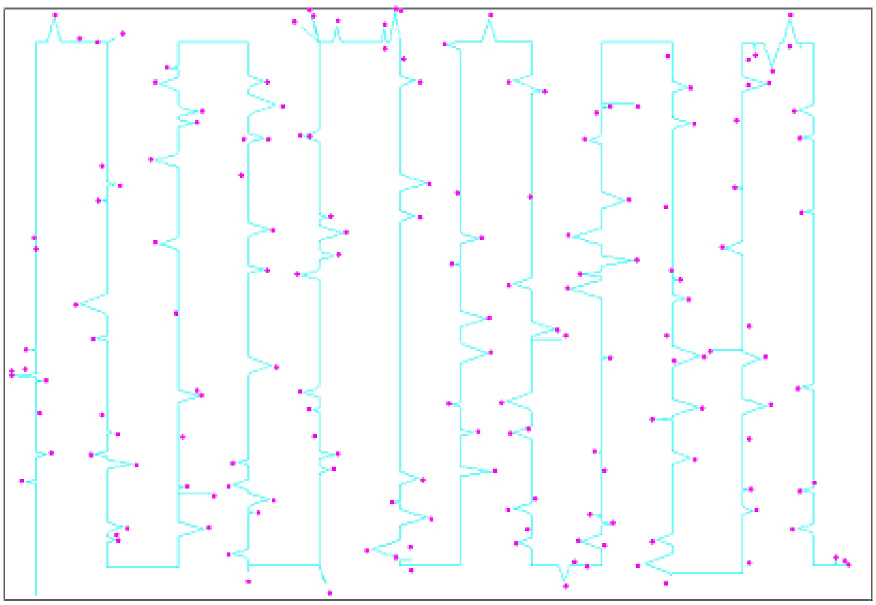

Figure 22: Planned Search with 20° as Departure Angle

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